Towards a Theory of Self-Segregation as a Response to Relative Deprivation: Steady-State Outcomes and Social Welfare

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Luigino Bruni and Pier Luigi Porta

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Oded Stark and You Qiang Wang

1. Introduction

People who transact individually in markets also belong to groups. Both the outcome of the market exchange and the satisfaction arising from the group affiliation impinge on well-being. But how and why do groups form and dissolve? The pleasure or dismay that arises from group membership can be captured in a number of ways and relative position is an appealing measure. A plausible response to transacting in a market that confers an undesirable outcome is to transact in another market (when the latter exists and participation in it is feasible). Labor migration is an obvious example. Similarly, one reaction to a low relative position in a given group could be a change in group affiliation. What happens then when people who care about their relative position in a group have the option to react by staying in the group or exiting from it?

We study this particular response in order to gain some insight into how groups form when individuals care about their relative position. To enable us to focus on essentials, we confine ourselves to an extremely stark environment. We hold the incomes of all the individuals fixed;[1] we restrict attention to a setting in which incomes are equally spaced; we start with all individuals belonging to a single group (exit is not an option) and then allow the formation of a second group (exit is feasible); and we allow costless movement between groups. We first use a payoff function that is the negative of the sum of the income differences between one individual and others in his group who have higher incomes. Next we use a payoff function that is the proportion of those in the individual's group whose incomes are higher than the individual's times their mean excess income. We derive stark and unexpected results. In the first case we find that the process converges to a steady-state equilibrium of individuals across groups wherein clusters of income sub-groups exist in each group. There is no unique cut-off point above or below which individuals move. In addition, the steady-state distribution differs from the steady-state distribution that would have obtained had group affiliation been chosen so as to maximize rank. In the second case we find that the process converges to a steady-state equilibrium wherein the individual with the highest income is alone in one group while all other individuals belong to the second group. Once again, the steady-state distribution is inconsistent with rank maximization. We characterize and explore the social welfare repercussions of the process.

Suppose there are two groups, A and B, and that the deprivation of an individual whose income is x arises only from comparisons with other individuals in his group; nothing else matters. We abstract from the intrinsic value of x. However, this is of no consequence whatsoever since x is retained (the individual's income is held constant) across groups. We are thus able to study group-formation behavior that is purely due to deprivation. The individual prefers to be affiliated with the group in which his deprivation is lower. When equally deprived (a tie), the individual does not change groups. The individual cannot take into account the fact that other individuals behave in a similar fashion. However, the individual's payoff, or utility, depends on the actions of all other individuals whose incomes are higher than his. A key feature of this situation is that tomorrow's group-selection behavior of every individual is his best reply to today's selection actions of other individuals. What will be the steady-state allocation of individuals across the two groups? What will be the allocation that minimizes the societal relative deprivation?

We employ two measures of relative deprivation. We motivate our use of these measures in Sections 2 and 3 below. Measuring social welfare as the inverse of the population's total relative deprivation, we find that while in both cases the level of social welfare associated with the steady-state distribution is higher than the level of social welfare that obtains at the outset, the steady-state allocations do not confer the maximal level of social welfare. Most interestingly, we also find that the allocation of individuals across the two groups that a welfare-maximizing social planner will choose is *identical* in the two cases. Thus while we admit a variance in perception and measurement and in the ensuing steady-state outcomes, we also point to a uniformity in policy design. From the perspective of a social planner this finding is of no trivial consequence. When a policymaker finds it difficult to unearth the precise manner in which individuals perceive relative deprivation, he could infer preferences from behavior: when there is a correspondence between observable steady states and hidden perceptions, policy analysts can await realization of the former to deduce the latter and then tailor their policy response to the inferred structure of preferences. Yet if the policy response to alternative structures of preferences happens to be invariant to these structures, awaiting realization of the steady states is not necessary and the policy intervention becomes more efficient.

Let there be a finite discrete set of individuals whose incomes are $x_1, x_2, \ldots, x_n$ where $x_1 \leq x_2 \leq \cdots \leq x_n$. In Section 2, the relative deprivation of an individual whose income is x_j and whose reference group consists of the n individuals is defined as $D(x_j) = \sum_{x_i > x_j} (x_i - x_j)$ and $D(x_j) = 0$ if $x_j \geq x_i$ for $i = 1, 2, \ldots, n$. In Section 3, the relative deprivation of an individual whose income is x_j is defined as $RD(x_j) = \sum_{i=j}^{n-1} [1 - P(x_i)](x_{i+1} - x_i)$ for $j = 1, 2, \ldots, n-1$ where $P(x_i) = Prob(x \leq x_i)$, and $RD(x_j) = 0$ if $x_j = x_n$. Note that both measures incorporate rank-related information beyond rank. In a population of two individuals, the rank of the individual whose income is 2 is the same regardless of whether the other individual's income is 3 or 30. However, both $D(\cdot)$and $RD(\cdot)$ duly differentiate between these two situations. Both measures imply that regardless of their distribution, all units of income in excess of one's own are equally distressing. As will be shown in Section 3, $RD(\cdot)$ further implies that a given excess income is more distressing when received by a larger share of the individual's reference group. ($RD(2)$ is higher in a population of two individuals whose incomes are 2 and 3 than in a population of three individuals whose incomes are 1, 2, and 3.)

2. The Steady-State Distribution when Relative Deprivation is Measured by $D(x_j)$

You board a boat in Guilin in order to travel on the Lijiang River. You can stand either on the port side (left deck) or on the starboard side (right deck) admiring the beautiful cliffs high above the banks of the river. Moving to the port side, you join other passengers, several of whom are taller than you. They block your view of the scenery. You notice that the starboard side is empty so you move there, only to find that other passengers who were disturbed by taller passengers have also moved to that side. You find your view blocked, which prompts you, as well as some other passengers, to return to the port side. And so on. Do these shifts come to a halt? If so, what will the steady-state distribution of passengers between the two decks look like? Will the steady-state distribution confer the best possible social viewing arrangement?

Incomes in the small region R where you live are fully used for visible consumption purposes. Any income (consumption) in your region that is higher than yours induces discomfort—it makes you feel relatively deprived. Another region, R', identical in all respects to your region except that initially it is unpopulated, opens up and offers the possibility that you, and for that matter anyone else, can costlessly move to R'. Who moves and who stays? Will all those who move to R' stay in R'? Will some return? And will some of those who return move once more? Will a steady-state distribution of the population across the two regions emerge? At the steady-state distribution, will the aggregate deprivation of the population be lower than the initial aggregate deprivation? Will it be minimal?

Consider a simple case in which there are ten individuals and individual i receives an income of i, $i = 1, \ldots, 10$. Suppose that initially all individuals $1, \ldots, 10$ are in group A. Group B just comes into existence. (For example, A can be a village, B—a city; A can be a region or a country, B—another region or country; and so on. In cases such as these we assume that the individual does not care at all about the regions themselves and that moving from one region to another is costless.) Measuring time discretely, we will observe the following series of migratory moves. In period 1, all individuals except 10 move from A to B because the deprivation of individual 10 is zero, while the deprivation of all other individuals is strictly positive. In period 2, individuals 1 through 6 return from B to A because every individual in region B except 9, 8, and 7 is more deprived in B than in A. When an individual cannot factor in the contemporaneous response of other individuals, his decision is made under the assumption of no group substitution by these individuals. In period 3, individual 1 prefers to move from A to B rather than be in A, and the process comes to a halt. Thus, after three periods, a steady state is reached such that the tenth and sixth through second individuals are in region A, while the ninth through seventh and first individuals are in group B. Figure 9.1 diagrammatically illustrates this example.[2]

What can be learned from this simple example? First, a well-defined rule is in place that enables us to predict group affiliation and steady-state

Period 0		Period 1		Period 2		Period 3	
Region *A*	Region *B*	Region *A*	Region *B*	Region *A*	Region *B*	Region *A*	Region *B*
10		10		10		10	
9			9		9		9
8			8		8		8
7			7		7		7
6			6	6		6	
5			5	5		5	
4			4	4		4	
3			3	3		3	
2			2	2		2	
1			1	1			1

Figure 9.1. The group-formation process and the steady-state distribution

distribution across groups. Second, until a steady state is reached, a change in group affiliation by any individual n is associated with a change in group affiliation by all individuals $i = 1, 2, \ldots, n - 1$. Third, the number of individuals changing affiliation in a period is declining in the (rank) order of the period. Fourth, the number of inter-group moves by individuals never rises with their income; individuals with low incomes change affiliations at least as many times as individuals with higher incomes. Fifth, the deprivation motive leads to a stratification steady-state distribution where clusters of income groups exist in each region rather than having a unique cut-off point above or below which individuals move. Sixth, the steady-state distribution differs from the distribution that would have obtained had group affiliation been chosen so as to maximize (ordinal) rank: under pure rank maximization the individual whose income is 3 would have ended up in B rather than in A.

Suppose that when equally deprived in A and B, the individual prefers A to B (an infinitesimal home preference). The steady state reached in this case differs from the steady state reached under the original assumption that when equally deprived (a tie) the individual does not migrate. Looking again at our example we will have the sequence shown in Figure 9.2. Interestingly, in the case of $(x_1, \ldots, x_n) = (1, \ldots, n)$ and an infinitesimal home preference, the number of periods it takes to reach the steady state is equal to the number of complete pairs in n, and the number of individuals who end up locating in A is $n/2$ when $n = 2m$, $(n - 1)/2$ when $n = 4m - 1$ or $(n + 1)/2$ when $n = 4m - 3$, where m is a positive integer.

Changing the incomes of all individuals by the same factor will have no effect on the pattern of migration. This homogeneity of degree zero property can be expected; when the payoff functions are linear in income

Period 0

Region A	Region B
10	
9	
8	
7	
6	
5	
4	
3	
2	
1	

Period 1

Region A	Region B
10	
	9
	8
	7
	6
	5
	4
	3
	2
	1

Period 2

Region A	Region B
10	
	9
	8
7	
6	
5	
4	
3	
2	
1	

Period 3

Region A	Region B
10	
	9
	8
7	
6	
	5
	4
	3
	2
	1

Period 4

Region A	Region B
10	
	9
	8
7	
6	
	5
	4
3	
2	
1	

Period 5

Region A	Region B
10	
	9
	8
7	
6	
	5
	4
3	
2	
	1

Figure 9.2. The migration process and the steady-state distribution with an infinitesimal home preference

differences, populations with income distributions that are linear transformations of each other should display the same migration behavior. Thus the propensity prompted by aversion to deprivation to engage in migration by a rich population is equal to the propensity to engage in migration by a uniformly poorer population. Migration is independent of the general level of wealth of a population.

Interestingly, the result of a non-uniform equilibrium distribution has already been derived, at least twice, in the very context that constitutes our primary example, that is, migration. Stark (1993, chap. 12) studies migration under asymmetric information with signaling. Employers at destination do not know the skill levels of individual workers—they only know the skill distribution. Employers are assumed to pay all indistinguishable workers the same wage based on the average product of the group of workers. Employers at origin, however, know the skill levels of individual workers and pay them a wage based on their marginal product. When a signaling device that enables a worker's skill level to be completely identified exists, and when the cost of the device is moderate, the equilibrium distribution of the workers is such that the least skilled migrate without investing in the signaling device, the most skilled invest in the signaling device and migrate, and the medium skilled do not migrate. Banerjee and Newman (1998) derive a qualitatively similar result. They study a developing economy that consists of two sectors: a modern, high productivity sector in which people have poor information about each other, and a traditional, low productivity sector in which information is good. Since from time to time individuals in both sectors need consumption loans that they may have difficulty repaying, collateral is essential. The superior information available in the traditional sector enables lenders to better monitor borrowers there as opposed to those in the modern sector. The superior access to credit in the traditional sector conditional on the supply of collateral, and the higher productivity in the modern sector prompt migration from the traditional sector to the modern sector by the wealthiest and most productive workers, and by the poorest and least productive employees. The wealthy leave because they can finance consumption on their own and do not need loans; the most productive leave because they have much to gain; and the poorest and the least productive leave because they have nothing to lose—they cannot get a loan in either location.

A crucial assumption of both Stark's and Banerjee and Newman's models is that information is asymmetric. So far, no migration study has analytically generated an equilibrium distribution of three distinct groups under symmetric information, nor has a migration study analytically generated an equilibrium distribution of more than three groups. As the present example yields an equilibrium distribution of more than three groups, and it does so under symmetric information, our example contributes to the theory of migration.

3. The Steady-State Distribution when Relative Deprivation is Measured by $RD(x_j)$

In earlier studies on relative deprivation and migration (Stark 1984, Stark and Yitzhaki 1988, and Stark and Taylor 1989, 1991) we drew largely on the writings of social psychologists, especially Runciman (1966), to formulate a set of axioms and state and prove several propositions, and we conducted an empirical inquiry. The measure of relative deprivation of an individual whose income is y, yielded by our analytical work for the case of a continuous distribution of income, is $RD(y) = \int_y^\infty [1 - F(x)]dx$ where $F(x)$ is the cumulative distribution of income in y's reference group. We have further shown that $RD(y) = [1 - F(y)] \cdot E(x - y \mid x > y)$: the relative deprivation of an individual whose income is y is equal to the proportion of those in y's reference group who are richer than y times their mean excess income. Our empirical work indicates that a distaste for relative deprivation, when relative deprivation is measured by RD, matters; relative deprivation is a significant explanatory variable of migration behavior.

Suppose there are n individuals and that individual i receives income i. Thus the configuration of incomes is $(1, \ldots, n-1, n)$. Suppose that initially all the individuals $1, \ldots, n-1, n$ are in region A. Region B opens up. (For example, migration restrictions are eliminated, or B comes into existence.) We measure time discretely.

Claim 1: If the configuration of incomes is $(1, \ldots, n-1, n)$, then the process of migration in response to relative deprivation reaches a steady state in just one period. Moreover, at the steady state, the individual with income n remains in region A while the rest of the population stays in region B.

Proof: It is trivial that in period 1 the individual with income n stays in region A while the rest of the population migrates to region B. Now consider the action of the individual with income i, where $i = 1, \ldots, n-1$. If the individual remains in region B, the individual's relative deprivation will be $(n-i)(n-1-i)/[2(n-1)]$.[3] If the individual returns to A, the individual's relative deprivation will be $(n-i)/2$. Note that $(n-i)(n-1-i)/[2(n-1)] < (n-i)/2$ for $i = 1, \ldots, n-1$. We thus have the result of the Claim. Q.E.D.

Corollary: Given the above setup and a real number $\alpha > 0$, the process of migration in response to relative deprivation will be identical in the two populations $P = \{1, \ldots, n-1, n\}$ and $P_\alpha = \{\alpha, \ldots, \alpha(n-1), \alpha n\}$.

Proof: The proof of the Corollary is a replication of the proof of Claim 1 since the two measures of relative deprivation in the proof of Claim 1 are multiplied by α, and therefore the inequality in the proof of Claim 1 carries through to the case of the Corollary. Q.E.D.

It follows that the propensity prompted by relative deprivation to engage in migration by a rich population is equal to the propensity prompted by relative deprivation to engage in migration by a uniformly poorer population. The pattern of migration is independent of the general level of wealth of the population.[4]

Note that the steady state is independent of whether individuals migrate simultaneously (as assumed) or in the order of their relative deprivation (with the most relatively deprived migrating first, the second most relatively deprived migrating second, and so on). In the latter case the steady state is reached after $n-1$ periods rather than in just one period.

The steady-state distribution differs from the distribution that would have obtained had group affiliation been chosen so as to maximize (ordinal) rank: under pure rank maximization the individuals with incomes $n-3, n-4, n-6, \ldots, n-(n-2)$ if n is an even number, and the individuals with incomes $n-3, n-4, n-6, \ldots, n-(n-1)$ if n is an odd number, would have ended up in region A rather than in region B.

Each of the two groups that form in the steady state is smaller than the original single group. It might therefore be suspected that migration is caused partly or wholly by an aversion to crowding. It is easy to see, however, that this is not so. When 1,000 individuals, each with income y, are in region A there is crowding but no migration; when ten individuals, five with income $y>1$ each and five with income $y-1$ each are in region A there is little crowding but much migration.

4. Societal Relative Deprivation and Social Welfare

Suppose we measure social welfare by the inverse of the population's total deprivation, where total deprivation is the sum of the deprivation of all the individuals constituting the population. It follows that social welfare is maximized when total deprivation is minimized. Consider first the case in which the payoff function is the negative of the sum of the income differences between one individual and others in his group who have higher incomes. While the social welfare associated with the steady-state distribution is higher than the social welfare associated with the initial period 0 allocation, individualistic group-formation behavior fails to produce maximum social welfare. The minimal total deprivation (TD) obtains when $(n, n-1, \ldots, i)$ are in A and $(i-1, i-2, \ldots, 1)$ are in B where $i=(n/2)+1$ if n is an even number and, as can be ascertained by direct calculation, where $i=(n+1)/2$ or $i=(n+3)/2$ when n is an odd number.[5]

Consider next the case in which the payoff function is the proportion of those in the individual's group whose incomes are higher than the individual's times their mean excess income. The steady-state allocation has n in region A and $(n-1, \ldots, 1)$ in region B. This allocation is Pareto efficient. However, the minimal total relative deprivation (*TRD*) obtains when $(n, n-1, \ldots, i)$ are in region A and $(i-1, i-2, \ldots, 1)$ are in region B where $i=(n/2)+1$ if n is an even number, and where $i=(n+1)/2$ or $i=(n+3)/2$ when n is an odd number.[6]

In both cases then, the policy response to the steady-state distributions attained by individuals who, while pursuing their own betterment, do not achieve a collectively preferred division is to distribute the population across the two regions in precisely the same manner.

As long as the number of different incomes is larger than the number of (reference) groups, total relative deprivation will not be minimized at zero. If there are as many groups as there are different incomes, total relative deprivation will be zero.

Adopting the perspective that social welfare is maximized when total relative deprivation is minimized is not as ad hoc as it may appear to be. Consider the following social welfare function: $SW=\bar{y}(1-G)$ where $\bar{y} = (\sum_{i=1}^{n} y_i)/n$ is income per capita in a society consisting of n individuals whose incomes are $y_1, y_2, \ldots, y_n$ and G is the Gini coefficient of income inequality. (It is easy to see that *SW* is higher upon an increase in any individual's income, and upon a transfer of any income from a high-income individual to a low-income individual.) It can be shown that $\left(\sum_{i=1}^{n} y_i\right)G = TRD$ where *TRD* stands for the total relative deprivation of the population.[7] Thus, *SW* can be rewritten as $SW=\bar{y}-(TRD/n)$: social welfare is the difference between income per capita and relative deprivation per capita. Since in the present setting incomes are kept intact, $\bar{y}$ is constant and *SW* is maximized when *TRD* is minimized.

We have implicitly assumed that region B is not subject to a capacity constraint: there is room in region B for the entire population but for one member. For the sake of concreteness, consider the case of an even n; of migration proceeding in the order of the intensity of relative deprivation; and of relative deprivation being measured by *RD*. We have seen that while individuals $1, 2, \ldots, n-1$ prefer to relocate to region B, it would be socially optimal to have only individuals $1, \ldots, n/2$ move there. Hence, if it so happens that region B can accommodate only up to one half of the population, migration will come to a halt precisely at a level that is socially optimal. We thus have an example in which a constraint on mobility is conducive to the attainment of maximal social welfare rather than constituting a hindrance to such an attainment.

5. Conclusions and Complementary Reflections

We have presented an analysis that contributes to the large and growing literature on the theory of non-market, social interactions pioneered by Schelling (1971, 1972) and recently added to, among many others, by Stark (1999), Glaeser and Scheinkman (2000) who provide a useful synthesis, and Becker and Murphy (2001).

We note that individuals belong to groups, clubs, neighborhoods, and various associations. When given a choice, individuals may want to revise their affiliation—form a new group, change their neighborhood, join another club, associate with others. Several considerations, both absolute and relative, impinge on these choices. In this chapter we have singled out for close scrutiny one such consideration—a distaste for relative deprivation. We have studied several repercussions when this measure is used as the exclusive determinant of affiliation.

We have assumed a given and uniform dislike of relative deprivation. Relative deprivation is a sensitive measure that encompasses rank-related information beyond mere rank. (It tells us that 1 compared to 3 is worse than 1 compared to 2, even though in both instances 1 ranks second.) An important question that is not addressed in this chapter is where the aversion to relative deprivation or, for that matter, the distaste for low rank, originates. Postlewaite (1998) argues that since over the millennia high rank conferred an evolutionary advantage in the competition for food and mating opportunities, the concern for rank is likely to be hard-wired (part of the genetic structure). More generally though, any setting in which rank impinges positively—directly or indirectly—on consumption ought to imply a concern for rank.[8] The study of why an aversion to relative deprivation exists and why individuals exhibit distaste for low rank invites more attention.

It is plausible to stipulate that the distaste for low rank will not be uniform across societies. Consequently, the extent of self-segregation across societies will vary. Since segregation is visible, whereas preferences are not, an inference may be drawn from the observed segregation to the motivating distaste, with more segregation suggesting stronger distaste.

We have shown that when individuals who initially belong to one group (costlessly) act upon their distaste for relative deprivation and self-select into any one of two groups, they end up splitting into two groups in a manner that is sensitive to the way in which relative deprivation is sensed and measured. However, when the social planner's response to a split is not sensitive to the way in which relative deprivation is conceptualized nor, for that matter, to the particular configuration of the split, there is no need to exert effort to unearth the specific configuration of the underlying motive or to await a particular manifestation of the behavior that the motive prompts.

We have described an endogenous process of voluntary segmentation into distinct groups; the division of the population into groups is not the outcome of an exogenous imposition of segregation. Assuming no comparisons between members of one group and another, we have shown that, as a consequence, aggregate relative deprivation is lowered. In broader contexts, the group partitioning could also be associated with improved social welfare as a result of reduced social tensions, fewer conflicts, less crime, and a mediated quest for status (as the inequality between those who compete with each other for status is reduced).

The opening of another region, *B*, facilitates shedding one's relative deprivation by allowing a group to split into two. Consider a reverse process, wherein regions *A* and *B* merge into a single composite region that constitutes everyone's reference group. In all cases (except the degenerate case in which all individuals have exactly the same income) the population's relative deprivation is bound to rise. Groups who are less well off in terms of absolute income will be better off in terms of well-being if they are allowed to secede, without any change in absolute income. Conversely, a group that is less well off in terms of absolute income that is forced to merge with a group that is better off in terms of absolute income becomes worse off. The pressure to form a separate state, for example, can be partially attributed to this aversion to relative deprivation; when such an aversion exists, the sole individual with less than 1 in *B* may prefer that option to having 1 in *A*, where 2 is present.

These considerations relate to federalism. The process of adding new members to a federation of nations usually draws on the expectation that in the wake of the integration, the incomes of the citizens of the new member nations will rise. The European Union, however, has taken great pains to ensure that the incomes of the citizens of the would-be member nations rise substantially *prior* to integration. Our approach suggests a rationale. To the extent that integration entails the formation of a new reference group, relative deprivation when 1 joins 2 would be reduced if $1\frac{1}{2}$ were to join 2, and would be eliminated altogether if 2 were to join 2.

The idea that externalities impinge asymmetrically on individuals' well-being and behavior has been with us for many years. Early proponents of this idea were of the opinion that the well-being of individuals rose in what they had and declined in what more prosperous people had. References of pioneering works that come readily to mind are Duesenberry (1949) who argued that individuals look up but not down when making comparisons, Stouffer *et al*. (1949) who, in spite of studying quite different behavior, independently argued likewise, and Davis (1966) who observed that in choosing higher performance career fields, which generally require graduate training, students in colleges and universities in the US were heavily influenced by their subjectively assessed relative standing in their college or university rather than by the subjective quality of the institution, *and* that they adjusted their career choices in a manner corresponding to their subjective (relative) standing in their college or university, tilting toward the low performance fields as their *relative* standing declined.[9] (As social psychologists, Stouffer *et al*. and Davis have carefully searched for the relevant set of individuals with whom comparisons are made—the reference group.) A recent manifestation of the asymmetric externalities idea takes the diametrically opposite view that while the utility of an individual rises in his own consumption, it declines in the consumption of any of his neighbors if that consumption falls below some minimal level; individuals are adversely affected by the material well-being of others in their reference group when this well-being is sufficiently lower than theirs (Andolfatto 2002). Our impression though is that in the course of the intervening five decades, the bulk of the theoretical work has held the view that individuals look up and not down, and that the evidence has overwhelmingly supported the "upward comparison" view.[10] (Helpful references are provided and reviewed in Frey and Stutzer (2002) and in Walker and Smith (2002).) The analysis in the preceding sections is in line with, and draws on this perspective. Nonetheless, it could be of interest to reflect on the manner in which our results will be affected if comparisons were to assume a symmetrical or quasi-symmetrical nature. It is easy to see why such a revised structure of preferences will not even yield a steady-state distribution to begin with. An example will suffice. Consider the first case and rewrite the payoff function of an individual whose income is x_j as follows: $D(x_j) = \sum_{x_i > x_j}(x_i - x_j) + \sum_{x_k < x_j}\alpha(x_k - x_j)$. Throughout this chapter we have assumed that $\alpha = 0$. Let us now have $\alpha > 0$, however small, retain the assumption of two regions, and consider the simplest case of $n = 2$. In this setting a steady state will never be reached: while 1 will want to separate from 2, 2 will want to stay with 1. There will be repeated and endless cycles. Let $\alpha = 1$ and consider the case of $n = 3$. Again, a steady state will not be reached and cycles will ensue: in period zero, 3, 2, and 1 are in A. In period one, 3 and 2 stay in A while 1 moves to B. (3 has the minimal sum of gaps (-3) which, if he were to move, would rise to (0); 2 has a sum of (0) and thus stands to gain nothing by moving; 1 has the sum of (3) which, upon a move, will be reduced to (0).) In period two, 3 and 2 move to B since each contemplates the move to result in a lowering of his period-one relative deprivation (from (-1) to (-2) and from (1) to (-1), respectively). But now 3, 2, and 1 are in B, which is the same configuration as that of period zero, prompting 1 to move to A, and so on. Alternatively, if we let $\alpha = -1$, implying that individuals seek to minimize the sum of absolute income gaps (in either direction), we will find once again, as can easily be verified, that a steady state will not be reached. The results obtained in this chapter constitute, therefore, a contribution to the study of group formation when affiliation choices are guided by an aversion to falling behind others, and when this aversion is modeled through particular measures that go beyond the crude measure of rank. The results are appealing both intuitively and analytically, and are consistent with a large body of theoretical and empirical literature.

Appendices

To differentiate between the cases that correspond to payoff functions $D(x_j)$ and $RD(x_j)$, we refer to total relative deprivation in the first case as *TD*, and to total relative deprivation in the second case as *TRD*.

Appendix 9.1

I

To find the division of a population of n individuals across groups A and B that confers the minimal total deprivation (*TD*) we proceed in two steps. First, given the size of the two groups, we show that the minimal *TD* is reached when high income individuals are in one of the groups and low income individuals are in the other group. (That is, the income of *any* individual who is in one group is higher than the income of *any* individual who is in the other group.) Second, given this distribution, we show that the minimal *TD* is reached when *half* of the individuals are in one group and the other half are in the other group.

Lemma: Let n be a fixed positive integer. Consider $\{a_1, a_2, \ldots, a_n\}$ where $a_1 < a_2 < \cdots < a_n$ and the a_i's are positive integers. Let $S(a_1, a_2, \ldots, a_n) = \sum_{1 \leq i,j \leq n} |a_i - a_j|$. Then $S(a_1, a_2, \ldots, a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1, 2, \ldots, n-1$.

Proof: For any $i < j$, we have $|a_i - a_j| = |a_j - a_{j-1}| + |a_{j-1} - a_{j-2}| + \cdots + |a_{i+1} - a_i|$.
Therefore, $|a_i - a_j| \geq j - i$ and $\left(\underset{\text{for all } i,j}{|a_i - a_j| = j - i} \right)$ if and only if $\left(\underset{\text{for } i=1,2,\ldots,n-1}{a_{i+1} = a_i + 1} \right)$. It follows that $S(a_1, a_2, \ldots, a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1, 2, \ldots, n-1$. (This minimum is $n(n^2 - 1)/3$.) Q.E.D.

Corollary: Consider the configuration of incomes $(1, \ldots, n-1, n)$. Let there be two groups, A and B, with $(i_1, i_2, \ldots, i_{n_A})$ in A, and $(j_1, j_2, \ldots, j_{n_B})$ in B, $n = n_A + n_B$. Let $TD = TD_A + TD_B$. Then, if n, n_A, n_B are fixed, TD reaches its minimum if and only if $(j_1, j_2, \ldots, j_{n_B}) = (1, 2, \ldots, n_B)$ or $(i_1, i_2, \ldots, i_{n_A}) = (1, 2, \ldots, n_A)$; that is,

either

Region A	Region B
n	
$\vdots$	
$n_B + 1$	
	n_B
	$\vdots$
	1

or

Region A	Region B
	n
	$\vdots$
	$n_A + 1$
n_A	
$\vdots$	
1	

Proof: Note that $TD_A = (1/2)\, S(i_1, i_2, \ldots, i_{n_A})$, $TD_B = (1/2)\, S(j_1, j_2, \ldots, j_{n_B})$. Thus, for fixed n_A, n_B, $\min TD_A \Leftrightarrow \min S(i_1, i_2, \ldots, i_{n_A})$, $\min TD_B \Leftrightarrow \min S(j_1, j_2, \ldots, j_{n_B})$. Assume that TD reaches its minimum at $(i_1^*, i_2^*, \ldots, i_{n_A}^*)$, $(j_1^*, j_2^*, \ldots, j_{n_B}^*)$. Without loss of generality, assume that $n \in (i_1^*, i_2^*, \ldots, i_{n_A}^*)$. Then, if $(i_1^*, i_2^*, \ldots, i_{n_A}^*) \neq (n_B + 1, \ldots, n)$, then $(j_1^*, j_2^*, \ldots, j_{n_B}^*) \neq (1, \ldots, n_B)$. By the Lemma, we have that $TD_A(i_1^*, i_2^*, \ldots, i_{n_A}^*) > TD_A(n_B + 1, \ldots, n)$, and $TD_B(j_1^*, j_2^*, \ldots, j_{n_B}^*) > TD_B(1, \ldots, n_B)$. Thus, $TD((i_1^*, i_2^*, \ldots, i_{n_A}^*), (j_1^*, j_2^*, \ldots, j_{n_B}^*)) > TD((n_B + 1, \ldots, n), (1, \ldots, n_B))$, which contradicts the assumption that TD reaches its minimum at $(i_1^*, i_2^*, \ldots, i_{n_A}^*), (j_1^*, j_2^*, \ldots, j_{n_B}^*)$. Hence, $(i_1^*, i_2^*, \ldots, i_{n_A}^*) = (n_B + 1, \ldots, n)$, and $(j_1^*, j_2^*, \ldots, j_{n_B}^*) = (1, \ldots, n_B)$. Conversely, by the Lemma, we have that $TD_A(i_1, i_2, \ldots, i_{n_A}) \geq TD_A(n_B + 1, \ldots, n)(or(1, \ldots, n_A))$, and $TD_B(j_1, j_2, \ldots, j_{n_B}) \geq TD_B(1, 2, \ldots, n_B)(or(n_A + 1, \ldots, n))$. Therefore, TD reaches its minimum at either of the two configurations. We have thus proved the Corollary. Q.E.D.

II

From the Lemma we know that the minimum of $S(a_1, a_2, \ldots, a_n)$ is $n(n^2-1)/3$. The total deprivation TD of $(n, n-1, \ldots, 1)$ is 1/2 of this minimum, that is, $TD = n(n^2-1)/6$. Let $n = n_A + n_B$, $n \geq 2$, $n_A \geq 1$. Then, by the Corollary, $TD_A = n_A(n_A^2-1)/6$, $TD_B = n_B(n_B^2-1)/6$. Therefore, $TD = (n_A(n_A^2-1)/6) + ((n-n_A)[(n-n_A)^2-1]/6) = (n^3 - 3n^2 n_A + 3nn_A^2 - n)/6$.

We seek to solve $\min_{1 \leq n_A \leq n} TD$. Since $(dTD/dn_A) = (1/6)(-3n^2 + 6nn_A)$ and $d^2TD/(dn_A)^2 = n > 0$, we have that the minimal TD obtains when $dTD/dn_A = 0$, that is, $n_A = n/2$. Therefore, if n is an even number, half of the n individuals will be in each of the two groups. With $TD_A = TD_B = n(n^2-4)/48$, $TD = TD_A + TD_B = n(n^2-4)/24$.

Appendix 9.2

I

Section I of Appendix 9.2 is identical to Section I of Appendix 9.1 except that *TD* in Appendix 9.1 is replaced by *TRD* in Appendix 9.2.

II

We next determine the size of the sub-groups that brings *TRD* to a minimum.

Let $(n, \ldots, i)$ be in region A, and let $(i-1, \ldots, 1)$ be in region B. Total relative deprivation in A is:[11]

$$TRD_A = \frac{1}{n-i+1} \cdot 1 + \frac{2}{n-i+1}\,\frac{1+2}{2} + \cdots + \frac{n-i}{n-i+1}\,\frac{1+2+\cdots+n-i}{n-i}$$
$$= \frac{1 + (1+2) + \cdots + (1+2+\cdots+n-i)}{n-i+1} = \frac{(n-i)(n-i+2)}{6}.$$

Total relative deprivation in B is:

$$TRD_B = \frac{1}{i-1} + \frac{2}{i-1}\,\frac{1+2}{2} + \cdots + \frac{i-1-1}{i-1}\,\frac{1+2+\cdots+i-1-1}{i-1-1}$$
$$= \frac{1 + (1+2) + \cdots + (1+2+\cdots+i-2)}{i-1} = \frac{i(i-2)}{6}.$$

Hence, $TRD = TRD_A + TRD_B = (1/6)[(n-i)(n-i+2) + i(i-2)]$. We seek to solve $\min_{1 \leq i \leq n} TRD$. Since $dTRD/di = (1/3)(-n + 2i - 2)$ and $d(TRD)^2/di^2 = 2/3 > 0$, we have that the minimal TRD obtains when $dTRD/di = 0 \Rightarrow -n + 2i - 2 = 0 \Rightarrow i = (n/2) + 1$. If n is an even number then the i that brings TRD to a minimum is $i^* = (n/2) + 1$, and, by direct calculation, $TRD = (1/12)(n^2 - 4)$. If n is an odd number, direct calculation yields that when $i = (n+1)/2$, $TRD = (1/12)(n^2 - 3)$, and that when

$i = (n+3)/2$, $TRD = (1/12)(n^2 - 3)$. Therefore, if n is an odd number, the i that brings TRD to a minimum is $i^* = (n+1)/2$ or $i^* = (n+3)/2$.

The result pertaining to the optimal split of the n individuals between the two regions can also be obtained by noting that for $(1, 2, \ldots, n)$, $TRD = (n^2 - 1)/6$. (This equation can be inferred, for example, from the expression above of $TRD_B = i(i-2)/6$ by setting $i - 1 = n$.) Let $n = n_A + n_B$, $n \geq 2$, $n_A \geq 1$. Then $TRD_A = (n_A^2 - 1)/6$ and $TRD_B = ((n - n_A)^2 - 1)/6$. Therefore, $TRD = (2n_A^2 + n^2 - 2n \cdot n_A - 2)/6$. We seek to solve $\min_{1 \leq n_A \leq n} TRD$. Since $dTRD/dn_A = (4n_A - 2n)/6$ and $d(TRD)^2/(d^2 n_A) = 4/6 > 0$, we have that the minimal TRD obtains when $dTRD/dn_A = 0 \Rightarrow 4n_A - 2n = 0 \Rightarrow n_A = n/2$. Therefore, if n is an even number, half of the n individuals will be in each of the two regions. With $TRD_A = ((n/2)^2 - 1)/6$ and $TRD_B = ((n/2)^2 - 1)/6$, $TRD = TRD_A + TRD_B = 2((n/2)^2 - 1)/6 = (1/12)(n^2 - 4)$.

Notes

1. When utility is derived both from absolute income and from relative income, and the utility function is additively separable, the difference in utilities across groups is reduced to the difference that arises from levels of relative income. Holding absolute incomes constant should not then be taken to imply that the individual does not care about his absolute income, and it enables us to study behavior that is purely due to considerations of relative income.
2. Since the myopic adjustment dynamics is deterministic, that is, the distribution in period t completely determines the distribution in period $t+1$, it follows that starting with everyone in A, the process will converge (if at all) to a unique steady state. To see this most easily, note that the richest individual will never move. Given the richest individual's immutable location, the second-richest individual has an optimal location and will need at most one period to get there. Given the stable location of the first two individuals, the third richest individual will have his own optimal location, which will be reached at most one period after the second individual has "settled down", and so on. No individual will have to move more times than his descending-order income rank. This reasoning assures us of convergence. As to uniqueness, allowing individuals to choose locations in a descending order of incomes well defines a path, and one path cannot lead to two destinations; the resultant "profile" is the only possible steady-state distribution.
3. In the case of $(x_1, \ldots, x_n) = (1, \ldots, n)$, $RD(x_j) = \sum_{i=j}^{n-1} (1 - i/n)$ (recall the last paragraph of Section 1). Since in this arithmetic series $a_1 = 1 - j/n$, $a_{n-j} = 1 - (n-1)/n$, and the number of terms is $n - j$, it follows that

$$RD(x_j) = \sum_{i=j}^{n-1} \left(1 - \frac{i}{n}\right) = \frac{\left(1 - \frac{j}{n} + 1 - \frac{n-1}{n}\right)(n-j)}{2} = \frac{n-j}{2n}(n - j + 1).$$

The relative deprivation of the individual with income i in region B can also be calculated by using this formula:

$$RD(i)|_{i\in B} = \frac{(n-1)-i}{2(n-1)}\left[(n-1)-i+1\right] = \frac{n-i}{2}\cdot\frac{n-1-i}{n-1}.$$

4. Note that the results of this section apply even if the population is multiplied by a natural number k. To see this, consider the configuration of incomes $\left(\underbrace{1,\ldots,1}_{k},\ldots,\underbrace{n,\ldots,n}_{k}\right)$. In period 1 the k individuals with income n stay in region A while the rest of the population migrates to region B. Now consider the action of an individual with income i, where $i=1,\ldots,n-1$. If the individual remains in region B, the individual's relative deprivation will be $(n-i)(n-1-i)/[2(n-1)]$ (as when $k=1$). If an individual with income i were to return to A, the individual's relative deprivation would be $(k/(k+1))(n-i)$. Since for any natural number k, $(k/(k+1))(n-i) > (n-i)\,(n-1-i)/2(n-1)$, the result of Claim 1 holds also for the case in which the population is multiplied by k.
5. The proof is in Appendix 9.1.
6. The proof is in Appendix 9.2.
7. The proof is Appendix 2 of Stark and Wang (2004).
8. In poor societies with meager assets, rank can serve as a proxy for collateral, making it easier for individuals to obtain credit.
9. Notably, students judged themselves by their "local standing" in their own college or university (that is, standing within their reference group) rather than across colleges or universities (that is, across reference groups). This self-assessment and the resulting response implied that being a "big frog in a small pond" or a "small frog in a big pond" mattered even when the absolute size of the "frog" did not change. Davis concluded that when parents who aspire for their son to opt for a higher-performance career field send their son to a "fine" college or university, "a big pond", they face a risk of him ending up assessing himself as a "small frog" thereby ending up not choosing a desirable career path.
10. For example, it has been argued that given the set of individuals with whom comparisons are made, an unfavorable comparison could induce harder work. This idea is captured and developed in the literature on performance incentives in career games and other contests. (Early studies include Lazear and Rosen (1981), Rosen (1986), and Stark (1990).) Loewenstein *et al.* (1989) provide evidence that individuals strongly dislike being in an income distribution in which "comparison persons" earn more. Clark and Oswald (1996) present evidence that "comparison incomes" have a significant negative impact on overall job satisfaction.
11. $\sum_{k=1}^{n}(1+2+\cdots+k) = \sum_{k=1}^{n}\frac{(1+k)k}{2} = \frac{1}{2}\sum_{k=1}^{n}k + \frac{1}{2}\sum_{k=1}^{n}k^2 = \frac{1}{2}\frac{(1+n)n}{2} + \frac{1}{2}\frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(n+2)}{6}$. Substituting $n-i$ for n yields the last expression of TRD_A.

References

Andolfatto, David (2002), "A Theory of Inalienable Property Rights", *Journal of Political Economy*, 110: 382–93.

Banerjee, Abhijit V., and Newman, Andrew F. (1998), "Information, the Dual Economy, and Development", *Review of Economic Studies*, 65: 631–53.

Becker, Gary S., and Murphy, Kevin (2001), *Social Markets: Market Behavior in a Social Environment*, Cambridge, Mass.: Harvard University Press.

Clark, Andrew E., and Oswald, Andrew J. (1996), "Satisfaction and Comparison Income", *Journal of Public Economics*, 61: 359–81.

Davis, James A. (1966), "The Campus as a Frog Pond: An Application of the Theory of Relative Deprivation to Career Decisions of College Men", *American Journal of Sociology*, 72: 17–31.

Duesenberry, James S. (1949), *Income, Savings and the Theory of Consumer Behavior*, Cambridge, Mass.: Harvard University Press.

Frey, Bruno S., and Stutzer, Alois (2002), *Happiness and Economics: How the Economy and Institutions Affect Human Well-Being*, Princeton: Princeton University Press.

Glaeser, Edward, and Scheinkman, Jose A. (2000), "Non-Market Interactions", Cambridge, Mass.: National Bureau of Economic Research, Working Paper 8053.

Lazear, Edward P., and Rosen, Sherwin (1981), "Rank-Order Tournaments as Optimum Labor Contracts", *Journal of Political Economy*, 89: 841–64.

Loewenstein, George F., Thompson, Leigh, and Bazerman, Max H. (1989), "Social Utility and Decision Making in Interpersonal Contexts", *Journal of Personality and Social Psychology*, 57: 426–41.

Postlewaite, Andrew (1998), "The Social Basis of Interdependent Preferences", *European Economic Review*, 42: 779–800.

Rosen, Sherwin (1986), "Prizes and Incentives in Elimination Tournaments", *American Economic Review*, 76: 701–15.

Runciman, Walter G. (1966), *Relative Deprivation and Social Justice: A Study of Attitudes to Social Inequality in Twentieth-Century England*, Berkeley: University of California Press.

Schelling, Thomas (1971), "Dynamic Models of Segregation", *Journal of Mathematical Sociology*, 1: 143–86.

—— (1972), "A Process of Residential Segregation: Neighborhood Tipping", in Anthony H. Pascal (ed.), *Racial Discrimination in Economic Life*, Lexington, Mass.: Lexington Books.

Stark, Oded (1984), "Rural-to-Urban Migration in LDCs: A Relative Deprivation Approach", *Economic Development and Cultural Change*, 32: 475–86.

—— (1990), "A Relative Deprivation Approach to Performance Incentives in Career Games and other Contests", *Kyklos*, 43: 211–27.

—— (1993), *The Migration of Labor*, Oxford and Cambridge, Mass.: Blackwell.

Stark, Oded (1999), *Altruism and Beyond: An Economic Analysis of Transfers and Exchanges within Families and Groups*, Cambridge: Cambridge University Press.

——and Taylor, J. Edward (1989), "Relative Deprivation and International Migration", *Demography*, 26: 1–14.

——(1991), "Migration Incentives, Migration Types: The Role of Relative Deprivation", *The Economic Journal*, 101: 1163–78.

——and Wang, You Qiang (2004), "On the Quest for Status as an Intervening Variable between the Distribution of Wealth and Growth", Manuscript, University of Bonn, Bonn.

——and Yitzhaki, Shlomo (1988), "Labour Migration as a Response to Relative Deprivation", *Journal of Population Economics*, 1: 57–70.

Stouffer, S. A., Suchman, E. A., Devinney, L. C., Star, S. A., and Williams, Jr., R. M. (1949), *The American Soldier: Adjustment during Army Life*. Princeton: Princeton University Press.

Walker, Iain, and Smith, Heather J. (2002), *Relative Deprivation: Specification, Development, and Integration*, Cambridge: Cambridge University Press.